

BLIND COST CRITERION TIMING RECOVERY

BACKGROUND OF THE INVENTION

5 Field of the Invention

The present invention relates to symbol timing recovery in a receiver of a digital communications system.

Cross-Reference to Related Applications

10 This application claims the benefit of the filing date of U.S. provisional application no. 60/242,306, filed on 10/20/00 as attorney docket no. 1043.001PROV.

Description of the Related Art

In many digital communications systems, a user generates digital information that is then processed into an encoded (e.g., error-correction encoded) and/or packetized stream of data. The stream of data is then divided into discrete blocks. Each of the blocks is mapped onto a corresponding one of a sequence of code or symbol values (“symbols”) chosen from a pre-defined alphabet A , and generated with a period T_s , sometimes referred to as the “baud” rate. Symbols may then be used to modulate an analog, e.g., radio frequency (RF) carrier, in amplitude, phase, and/or frequency prior to physical transmission through the communication medium. Many methods of mapping exist and are well known in the art, and these pre-defined alphabets are generated based on certain criteria. For example, data may be mapped into symbols of a complex data stream as pairs of in-phase (I) and quadrature phase (Q) component values. I and Q component values of the complex data stream may then be used to modulate cosine and sine components of a quadrature oscillator that are subsequently upconverted to an RF carrier. Modulation formats such as quadrature amplitude modulation (QAM) and vestigial sideband amplitude modulation (VSB) are common formats used for transmission of digital television

signals in accordance with, for example, the ATSC standard for digital television, "ATSC Digital Television Standard," Doc. A/53, Sept. 1995.

The modulated carrier signal transmitted through the medium (e.g., wire, optical fiber, atmosphere, space, magnetic recording head/tape, etc.) comprises a series of analog pulses, each 5 analog pulse being amplitude or phase modulated by a corresponding symbol in the sequence. The pulse shape used typically extends many symbol periods in time. This introduces the possibility of adjacent pulses corrupting each other, a phenomenon known as inter-symbol interference (ISI).

As is known in the art, transmit and receive filters may be selected to minimize the effects 10 of ISI. A pulse shape is selected so as to provide i) a high amplitude signal at or near the sampling instant, ii) a rapid rate of decay past the sampling instant, iii) a zero-value at integer multiples of the baud rate, and iv) a realizable (or very closely approximate) implementation. Thus, the pulse shape is selected such that it obeys the well known "Nyquist pulse shaping 15 criterion for zero ISI" as stated in condition (iii). As is known in the art, transmit and receive filters are usually selected so that when they are in cascade in the signal path a desired pulse shape is produced for a detector. For example, a commonly used filter that provides a "Nyquist pulse" may have its impulse response selected from the raised cosine family of functions. The transmit and receive filters are then selected as root-raised cosine filters. FIG. 1A shows a sequence of three raised cosine (Nyquist) pulses 101, 102, and 103, the first two instances 20 corresponding to a bit value and the third instance corresponding to a complement bit value. The symbol repetition period, T_s , is the time between the largest, unity-valued peaks of two consecutive pulses in the sequence.

FIG. 1A shows three consecutive instances of a raised cosine pulse with ideal sampling (sampling period equals the symbol repetition period T_s and phase $\tau=0$). In FIG. 1A, when the 25 pulse is sampled at top-center of the unity-valued peaks, the amplitude measured at time equal 30 (arbitrary time base), is the sum of amplitudes of the three pulses, or $a+b+c$. Note that the contribution from adjacent pulses is zero (because $b=c=0$), so no ISI effect is introduced. In general, with ideal sampling, the signal contribution of other pulses is zero at non-zero, integer

multiples of the symbol repetition period T_s . Ideal sampling of the sequence of pulses, therefore, results in zero ISI.

FIG. 1B shows these same three consecutive instances of a raised cosine pulse with one form of non-ideal sampling (e.g., sampling with period equal to symbol period T_s and sampling phase $\tau \neq 0$). In FIG. 1B, when the pulse is sampled at time equal 28, the amplitude measured is again the sum of amplitudes of the three pulses, or $a+b+c$. However, unlike the sampling of FIG. 1A, the signal contribution of the other pulses, b and c , is now non-zero even though the sampling period is still T_s . This illustrates how non-ideal sampling introduces ISI effects.

Thus, it is desirable for a receiver to sample a received signal of consecutive pulses at nearly ideal sampling instances. When nearly ideal sampling is achieved, the signal is said to be sampled at “top dead center” of the symbol period, which also represents the “ideal sampling phase.” Many factors combine to make this task difficult for a receiver, including the unknown propagation delay from the transmitter to the receiver, possible mismatch in the oscillator frequencies in the transmitter and receiver (causing the relative delay to drift over time), and multipath signal interference. For these reasons, a receiver both estimates and tracks the relative timing offset, which is referred to in the art as “timing recovery” and/or “symbol/baud synchronization.”

A receiver performs several functions to demodulate and decode a received signal. Receiver functions include, for example, tuning and RF demodulation of the received signal to an intermediate frequency (IF) signal; synchronization of the carrier loop to the RF carrier; symbol timing recovery (baud synchronization); sampling according to the baud rate or symbol period; equalization; symbol detection; and decoding. After RF demodulation, the received signal is sampled by, for example, an analog-to-digital (A/D) converter. Timing recovery attempts to both detect the symbol repetition period T_s and synchronize sampling instances to the top-dead-center of the pulse shapes. The timing recovery system then tracks variations in the detected period of T_s . A subsequent detector examines each sample to generate either a soft or hard decision for the symbol that corresponds to the sample. The present invention is concerned with the timing recovery function of the receiver.

Many methods exist in the art for timing recovery. One method uses a separate pilot tone

in phase with the modulation process that is transmitted in addition to the information-bearing signal. The receiver derives the symbol timing information from the pilot tone. However, including a reference timing signal reduces channel throughput (and uses both extra power and bandwidth) for pilot tone transmission and reception. Consequently, many applications use blind 5 techniques for symbol timing, and also equalization.

With blind techniques, timing information is derived directly from the received signal itself. An “error tracking synchronizer” continually estimates and tracks the timing error present, and adjusts a locally generated timing reference responsive to the error estimate. A “feedforward synchronizer” does not continually adjust a locally generated reference based on an error 10 estimate, but instead processes the received signal to directly generate the reference.

Feedforward synchronizers are often employed in burst-mode communication systems. In either error tracking or feedforward synchronizers, a “decision directed” synchronizer utilizes the receiver’s estimates of the transmitted symbol values to synthesize the timing estimate. The synchronizer is termed a “non-data-aided” synchronizer, since no data is transmitted without user 15 content to aid in timing recovery.

Synchronizers may operate in continuous time, discrete time, or a combination of both continuous and discrete time. Continuous time synchronizers apply the reference signal generated by the synchronizer to the clock of the A/D converter, thereby adjusting the actual sampling period of the analog input signal. Conversely, discrete time synchronizers leave the 20 A/D converter in a “free-running” mode and apply the timing reference to a digital interpolator that adjusts the phase of the digital sample sequence.

FIG. 2 shows a block diagram of a prior art timing recovery system 200 that may be employed in a receiver. Timing recovery system 200 receives sample sequence $y[n]$ from A/D converter 201 coupled to demodulator 222. A/D converter 201 is in “free-running” mode and is governed by a free running oscillator 202. The sample sequence $y[n]$ is not synchronized to the symbol phase τ_s or period T_s . Timing recovery module 200 includes digital interpolator 203 that 25 adjusts the phase τ and period T of the digital sample sequence to generate $y[nT+\tau]$ (also labeled herein as $y_n(\tau)$). Interpolator 203 may be implemented with a poly-phase filter. The interpolated sequence $y_n(\tau)$ is in relatively close synchronization with the symbol period T_s and phase τ_s .

The interpolated sequence $y_n(\tau)$ is passed to timing phase detector 204. Timing phase detector 204 generates an estimate of the timing phase error (termed herein as “phase error” e_τ) that represents the difference between the actual sampling phase τ and the ideal sampling phase τ_s . The phase error e_τ is then filtered with loop filter 205 to reject high frequency components of
5 the signal and integrate phase over time to adjust frequency. Local timing reference 206, shown in FIG. 3 as a numerically controlled counter (NCC), uses the filtered version of error estimate e_τ from loop filter 205 to adjust phase and/or period of the timing reference signal used to drive sampling of the signal. The reference of local timing reference 206 controls interpolator 203 in a way that tends to drive the phase error e_τ toward a fixed mean point, typically set as zero.

10 An alternative embodiment for a prior art timing recovery system 300 is shown in FIG. 3, in which the phase error e_τ is used to adjust the sampling clock on A/D converter 301 directly. The sampled sequence $y_n(\tau)$ is passed through timing phase detector 204 which generates an estimate of phase error e_τ in a manner similar to that of FIG. 2. The phase error e_τ is filtered with loop filter 205 and provided to local timing reference 306. Local timing reference 306, shown in
15 FIG. 3 as voltage controlled oscillator (VCO), uses the filtered phase error e_τ from loop filter 205 to adjust phase and/or period of its output reference signal. The reference signal of local timing reference 306, in turn, controls A/D converter 301 in a way that tends to drive the phase error e_τ toward a fixed mean point, typically set as zero.

20 Many methods exist in the prior art that may be employed by timing phase detector 204 to calculate the error estimate e_τ from the sequence $y_n(\tau)$. One such technique defines a cost criterion (also referred to as a cost function) that is a function of the timing phase τ . Timing phase τ is adjusted to a value that minimizes the cost function by a gradient descent technique. The value of τ which minimizes the cost function also causes the derivative of the cost function
25 with respect to τ (also known as the “gradient function”) to be zero. Thus, the desired timing phase may be found by adjusting, or stepping, the value of τ in a direction opposite to the sign of the cost function (i.e., mathematically, the trajectory of the parameter τ descends the steepest slope of the cost function). Therefore, this approach is sometimes termed a gradient descent strategy.

These prior art methods generally calculate the gradient of a mean squared error (MSE)

function. The MSE cost function J_{MSE} is defined as the expected value of the square of the difference between a received digital sample $y_n(\tau)$ and the actual transmitted symbol s as in equation (1):

$$J_{MSE} = E[|y_n(\tau) - s|^2] \quad (1)$$

5 where $E[\bullet]$ denotes the mathematical expectation, or “expected value of” “ \bullet .”

The derivative of J_{MSE} with respect to τ , $dJ_{MSE}/d\tau$, may be used as the phase error e_τ , and may be written as in equation (2):

$$dJ_{MSE}/d\tau = (dJ_{MSE}/dy_n(\tau))dy_n(\tau)/d\tau. \quad (2)$$

where the first term on the right-hand side, $(dJ_{MSE}/dy_n(\tau))$, is the derivative of J_{MSE} with respect

10 to $y_n(\tau)$, which is proportional to $|y_n(\tau) - s|$. For typical implementations, the expectation operator is omitted and instantaneous values are used by the process instead. The derivative $dJ_{MSE}/dy_n(\tau)$ is the least mean square (LMS) error term and is defined as $e_{LMS}[n]$ in the timing phase detector error of equation (2'):

$$dJ_{MSE}/d\tau = e_{LMS}[n] dy_n(\tau)/d\tau. \quad (2')$$

15 where $e_{LMS}[n] = |y_n(\tau) - s|$. Computation of $dy_n(\tau)/d\tau$ may be approximated using a finite impulse response (FIR) filter, such as described in Lee and Messerschmitt, Digital Communication, Appendix 17-B, Kluwer Academic Publishers, Norwell, MA, Second Edition, 1994, which is incorporated herein by reference.

The error term $e_{LMS}[n]$ depends on the transmitted symbol value s . Actual symbol values
20 may be available as an acquisition aid to the receiver during a training period. However, if either no training interval is defined in the system or the training period is insufficient for reliable acquisition, the receiver may use its estimates of the transmitted symbols instead (referred to as “decision directed” mode). These decision-directed systems are examples of blind signal systems, since they process a received signal without knowledge of the actual transmit symbol
25 information. Under these conditions, before adequate convergence of the timing loop, the estimates of the transmitted symbols are prone to error, making decision directed adaptation unreliable. When the system has stabilized using some other acquisition method, the receiver then switches to the decision-directed gradient method for tracking.

Many blind techniques exist for timing recovery based on the demodulated signal. For example, some systems may use a timing phase detector which obtains a timing estimate from both high-pass and low-pass filtered versions of the demodulated signal. Such a system is disclosed in U.S. Patent no. 5,872,815 to Strolle et al., entitled Apparatus for Generating Timing

5 Signals for a Digital Receiver," which is incorporated herein by reference.

A receiver also generally applies equalization to the sample sequence prior to forming hard decisions for symbols from the received sample sequence. Equalization is used to reduce the effects of ISI, caused by phenomena such as i) residual timing error (for example as in FIG. 1B), ii) multipath distortions from the propagation channel, and/or iii) approximations to the 10 ideal transmit and receive filters for ease of implementation. As with timing recovery, the samples representing the received symbols are filtered by equalizer coefficients, which are adjusted to minimize a cost function.

One such blind cost criterion employed for equalization is the constant modulus (CM) criterion. The stochastic gradient descent of the CM criterion for equalization is known as the 15 Constant Modulus Algorithm (CMA). The CMA algorithm is described in an article by D.N. Godard entitled "Self-Recovering Equalization in Two-Dimensional Data Communication Systems," *IEEE Transactions on Communications*, vol. 28, no. 11, pp. 1867-1875, Oct. 1980, which is incorporated herein by reference. The CM criterion and CMA algorithm were further developed to de-couple equalization and carrier recovery functions in a receiver. Such use of 20 CM criterion and CMA algorithm for equalization is described in J.R. Treichler et al., "A New Approach to Multipath Correction of Constant Modulus Signals," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. ASSP-31, no. 2, April 1993, which is incorporated herein by reference. Systems that use such CMA algorithm for adaptive 25 equalization, such as that described in U.S. Patent No. 5,872,815 to Strolle et al., do not employ the CM criterion or its variants for timing recovery.

SUMMARY OF THE INVENTION

The present invention relates to symbol timing recovery employing a blind cost criterion of Bussgang-class cost functions, such as the Constant Modulus (CM) criterion, and its 30 derivative with respect to timing offset, in order to generate a timing phase error estimate. This

estimate is then used to adjust the period and phase of the sample sequence toward the period and phase of the transmitted symbols, driving the timing phase error toward a point, such as zero in the mean. The sample values used may be either i) samples themselves, ii) processed (e.g., interpolated) samples, or iii) equalized and processed samples. In addition, timing phase error estimates for other cost criteria, including the least mean squares algorithm, may be generated. These timing phase error estimates are selected either alone or in combination for deriving the adjustment of the period and phase of the sample sequence. In addition, methods of blind timing recovery are described for VSB signals based on a single-axis CM (SA-CM) criterion which use real part extraction. Further, the present invention may be extended to jointly adjust timing phase and equalizer coefficients.

In accordance with an exemplary embodiment of the present invention, timing recovery of symbols in a received signal comprises (a) generating a sequence of samples from the received signal with a sample period and sample phase related to a symbol rate of the symbols; and (b) generating a phase error for a current sample from the sequence based on a gradient of a blind cost criterion of Bussgang-class cost functions. At least one of the sample period and sample phase are repetitively adjusted based on the phase error such that a magnitude of the phase error is driven to a predetermined point. Further embodiments calculate a blind cost error term based on the sample by forming an approximation of a derivative of the received signal with respect to the sampling phase; and combining the blind cost error term and the approximation to form the phase error.

BRIEF DESCRIPTION OF THE DRAWINGS

Other aspects, features, and advantages of the present invention will become more fully apparent from the following detailed description, the appended claims, and the accompanying drawings in which:

FIG. 1A shows three consecutive instances of a raised cosine pulse when ideal sampling is employed;

FIG. 1B shows three consecutive instances of a raised cosine pulse when non-ideal sampling is employed;

FIG. 2 shows a block diagram of a prior art timing recovery module;

FIG. 3 shows a block diagram of an alternative prior art timing recovery module;

FIG. 4 shows a broadcast communication system including a receiver with symbol timing recovery in accordance with an exemplary embodiment of the present invention;

5 FIG. 5A illustrates a constant modulus criterion based on determining a radius about the origin of an 8-PSK source constellation;

FIG. 5B illustrates a constant modulus criterion based on determining a radius about the origin of a 16-QAM source constellation;

10 FIG. 6 shows an exemplary embodiment of a timing recovery system in accordance with the present invention;

FIG. 7 shows a first exemplary implementation of the timing phase detector of FIG. 6;

FIG. 8 shows a second exemplary implementation of the timing phase detector of FIG. 6;

FIG. 9 shows a third exemplary implementation of the timing phase detector of FIG. 6;

FIG. 10 shows a fourth exemplary implementation of the timing phase detector of FIG. 6;

15 FIG. 11 shows a weighted averaging circuit that may be substituted for the multiplexer in FIGS. 8 and 10;

FIG. 12 shows the timing phase detector of FIG. 7 adapted for use in a vestigial sideband (VSB) receiver; and

20 FIG. 13 shows a fifth exemplary implementation of the timing phase detector of FIG. 6 switching between gradient-based and non-gradient based timing phase error detectors.

DETAILED DESCRIPTION

FIG. 4 shows a digital communication system 400 including receiver section 403 that employs symbol timing recovery and equalization in accordance with exemplary embodiments of the present invention. Communication system 400 comprises transmitter section 401 transferring signals through transmission medium 402 to receiver section 403. Digital communication

system 400 may be employed to broadcast digital television signals in accordance with, for example, the ATSC standard for digital television, "ATSC Digital Television Standard," Doc. A/53, Sept. 1995.

Transmitter section 401 comprises digital encoding system 411, pulse shaper 412, RF upconverter 413, and transmit antenna 414. Transmitter section 401 receives user data from information source 410 (such as video, audio, or computer files) coupled to digital encoding system 411. Digital encoding system 411 may provide analog-to-digital (A/D) conversion, error-correction encoding, and bit-to-symbol mapping to generate a sequence of symbols selected from a predetermined alphabet. For example, data may be mapped into a complex-valued signal stream with pairs of in-phase (I) and quadrature-phase (Q) components. Digital encoding system 411 provides the symbols to pulse shaper 412. Pulse shaper 412, which may be implemented with a shaping filter, provides pulse shaping of the symbols. The signal generated by pulse shaper 412 is provided to RF upconverter 413 that i) uses the signal to modulate a radio frequency (RF) carrier and ii) amplifies the signal.

Various modulation techniques, such as quadrature amplitude modulation (QAM), m -ary phase-shift keyed modulation (m -ary PSK), or vestigial sideband modulation (VSB) known in the art of digital television may be employed to modulate the carrier. Other modulation techniques include m -ary offset QAM, modulation in accordance with the CCITT 802.11 standard, and V.27 modulation. For example, a quadrature oscillator may be employed as an RF modulator and use i) the I signal component to modulate the cosine component generated by the oscillator and ii) the Q signal component to modulate the sine component of the oscillator. The modulated and amplified RF carrier is then emitted into the transmission medium 402 as an RF signal via transmit antenna 414.

Receiver section 403 comprises receive antenna 420, tuner 421, demodulator 422, timing recover module 423, detector 424, and decoder 425. Antenna 420 receives the RF signal from the transmission medium 402. The received RF signal may have added phase and amplitude noise, doppler shift, multipath effects, distortion, dispersion, and/or other effects typically associated with the particular transmission medium.

Tuner 421 isolates the RF signal of interest received through antenna 420 from medium

402 and translates the signal, in frequency, to an intermediary frequency (IF) signal (“passband signal”). Many different techniques exist in the art for demodulating a modulated RF carrier to provide a near-baseband or passband signal, and some of these systems may not necessarily employ digital signal processing techniques. One skilled in the art would realize that the process 5 of receiving, isolating, and demodulating the transmitted signal as described herein is described as an aide to the description of symbol timing recovery and synchronization in accordance with exemplary embodiments of the present invention.

Demodulator 422 processes the passband signal as follows. The received signal is sampled via an analog-to-digital (A/D) converter 450, creating a digital sample sequence. A/D 10 converter 450 may generate sample values at a rate greater than the symbol rate T_s . As would be apparent to one skilled in the art, decimation and/or interpolation may be employed at various points within receiver section 403 to generate sample sequences at different sampling rates and different sampling phases. Demodulator 422 may apply pre-filter/power detector 451 to the digital sample sequence to reduce noise effects and to generate a power level estimate for 15 automatic gain control (AGC). Gain of the near-baseband signal is adjusted via an automatic gain control (AGC) circuit 452. Gain of the IF signal, and/or RF signal may also be controlled via AGC circuits implemented similarly to that of AGC circuit 452. AGC circuit 452 controls dynamic range of the near-baseband signal to within a predetermined range. Complex demodulator 453 generates the near-baseband signal from the passband signal, and splits the 20 near-baseband signal into a complex sample sequence of in-phase (I) and quadrature-phase (Q) components. The near-baseband signal is not at precise baseband due to clock inaccuracies, oscillator phase noise, or doppler shift.

FIG. 4 and subsequent figures show the signal path of I and Q components (and processing) as a single line, though they may be conveyed and/or processed through separate 25 paths. In general, signals processed by receiver section 403 are complex-valued, though for some applications only I components are used. For example, with VSB signaling, the Q component is the Hilbert transform of the I component and thus contains no additional user information. In addition, while the figures illustrate symbol timing recovery in a digital television receiver, many components in addition to those shown may exist, but these additional components are not 30 necessary to understanding the present invention and are omitted for simplicity.

Timing recovery module 423 detects and generates an estimate of timing phase error e_τ . Phase error e_τ is derived from the derivative of a blind cost criterion in accordance with present invention. Timing recovery module 423 aligns or otherwise synchronizes i) the period T and phase τ for sampling of the received signal to ii) the symbol period (baud rate) T_s and phase τ_s of 5 I and Q signal components of the transmitted signal. Timing recovery module 423 controls the period T and phase τ of the sample sequence such that the phase error e_τ is driven toward a desired point. For example, the phase error e_τ is driven in mean toward zero. One skilled in the art would realize that such control may drive toward a non-zero point, and may be driven in mean, or similar type of measure. Timing recovery module 423 thus allows for generating a 10 sequence of samples from the received analog signal for which sampling occurs at the symbol period and at the peak/center of the pulse representing each symbol.

The sequence of samples is then provided to detector 424, which generates an estimate for each sample corresponding to a transmitted symbol. Detector 424 filters the signal with matched filter 460 whose frequency response characteristic matches the frequency response of 15 pulse shaper 412. For example, if the pulse shaper applies a filter corresponding to a root-raised cosine function, the matched filter also applies a root-raised cosine to i) create a Nyquist pulse at the matched filter output and ii) maximize signal-to-noise ratio (SNR).

Since transmission medium 402 may distort the signal waveform passing through it, each sample from the timing recovery module may include ISI effects from adjacent samples, even 20 though the sample values are generated at instances corresponding to the correct symbol period. Equalizer 461 of detector 424 applies equalization to the sequence of samples from timing recovery module 423. Equalizer 461 adjusts the sample values to remove distortion effects and restore the “zero ISI” characteristics of the sample sequence. Equalizer 461 may be implemented with an adaptive, multi-tap filter in a linear or non-linear architecture with a finite or infinite 25 impulse response. Detector 424 may also include a phase-locked loop (PLL) 462 for residual carrier loop recovery and synchronization. Carrier loop recovery and synchronization translates the signal to precise baseband by compensating for a residual offset between the carrier frequency of the transmitted signal and the frequency selected by tuner 421. While shown as part of detector 424, either detector 424 or decoder 425 may employ, for example, slicer 463 to 30 generate a hard decision for each symbol from the corresponding sample.

Decoder 425 processes the sequence of symbol decisions to reverse the various levels of encoding (symbol-to-data mapping, error correction, re-formatting, and/or depacketizing) to reconstruct the original user information. For example, decoder 425 may include a QAM or VSB decoder, as well as Reed-Soloman error correction to drive the probability of error below a 5 predetermined threshold. Re-formatting may include decoding in accordance with a digital television standard (e.g., MPEG-1, MPEG-2).

In accordance with exemplary embodiments of the present invention described herein, symbol timing recovery employs the Constant Modulus (CM) criterion and its stochastic gradient descent, termed the CM algorithm (CMA), to generate the phase error e_τ using blind adaptation.

10 This timing phase error estimate is used to control the frequency (period) and phase of the sample sequence either by 1) adjusting the sample clock in the A/D converter, or 2) interpolating the digital sample sequence such that the phase error e_τ is driven toward zero. While the preferred embodiments of the present invention are described herein with respect to the CM criterion and the CMA algorithm, one skilled in the art would realize that the present invention is not so 15 limited. Other blind cost criteria of Bussgang-class cost functions may be used that are related to the CM criterion and CMA algorithm described herein. The Bussgang-class of functions are well-known in the art and are described in, for example, Simon Haykin, Blind Deconvolution, Chapter 2, PTR Prentice-Hall, Englewood Cliffs, NJ (1994), whose teachings are incorporated herein by reference.

20 The CM criterion penalizes the deviation of the dispersion of the magnitude squared of the received signal from a pre-calculated constant referred to as the “dispersion constant” or the “Godard radius.” FIGs. 5A and 5B illustrate that the constant modulus criterion is based on deviation from a “radius” about the origin of, for example, a source constellation.

FIG. 5A shows a radius 501 of an 8-PSK (phase-shift keyed) constellation plotted for real 25 (e.g., Re or I) and imaginary (e.g., Im or Q) components. In FIG. 5A, each point (symbol) lies on the circle defined by this radius (termed a constant modulus system), and the CM criterion penalizes distance of a received symbol (e.g., symbol 502) from this circle. Even though the constellation may rotate, the constellation remains on the circle, and so applying a CM criterion to this constellation does not penalize spatial rotation of the constellation due to residual carrier

offset. FIG. 5B shows a 16-QAM (quadrature amplitude modulation) constellation plotted for real and imaginary components. In FIG. 5B, groups of points (symbols) lie on corresponding concentric circles 511, 512, and 513. The CM criterion determines a radius 514 of circle 515, which is not necessarily a radius of one of the concentric circles 511, 512, and 513 (termed non-
5 constant modulus), as a “common” radial distance from the origin for the points of the constellation. As with the constellation of FIG. 5A, the CM criterion penalizes distance of a received symbol (e.g., symbol 503) from this circle 515.

The CM criterion defines a cost function J_{CM} that may be expressed as given in equation (3):

$$10 \quad J_{CM} = E[(\rho^2 - |y_n(\tau)|^2)^2] \quad (3)$$

where ρ^2 is the dispersion constant (Godard radius), $y_n(\tau)$ is the discrete value that represents the sampled signal (e.g., the sampled symbol), and τ represents the timing (sampling) phase. The dispersion constant ρ^2 is a quantity that can be determined from the type of modulation employed (e.g., QAM, BPSK, etc.). The dispersion constant ρ^2 may be derived by calculation, experiment, or a combination of both for a particular implementation. Note that if y_n is real-valued, then the modulus operator in equation (3) can be omitted.
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The derivative of J_{CM} with respect to τ may be expressed as in equation (4):

$$dJ_{CM}/d\tau = (dJ_{CM}/dy_n(\tau))dy_n(\tau)/d\tau. \quad (4)$$

The derivative of J_{CM} with respect to $y_n(\tau)$ is proportional to that given by (5):

$$20 \quad dJ_{CM}/dy_n(\tau) = E[y_n(\tau)(\rho^2 - |y_n(\tau)|^2)]. \quad (5)$$

The phase error e_τ is formed by removing the expectation operator and combining (4) and (5)

$$e_\tau = dJ_{CM}/d\tau = e_{CMA}[n] dy_n(\tau)/d\tau \quad (4')$$

where $e_{CMA}[n] = y_n(\tau)(\rho^2 - |y_n(\tau)|^2)$.

In equations (4), (4'), and (5), $y_n(\tau)$ represents the discrete value for the sampled signal, and the sequence discrete values may have various levels of pre-processing applied. Thus, $y_n(\tau)$ may either be the original sample sequence, interpolated data from the original sample sequence, or the equalized sample sequence (either original or interpolated). Note that if $y_n(\tau)$ is taken as
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the equalizer output sample, then $e_{CMA}[n]$ is the CMA error term that is used to update the equalizer coefficients. The CMA error term used to update the equalizer coefficients may be as shown by D.N. Godard in, "Self-Recovering Equalization in Two-Dimensional Data Communication Systems," IEEE Transactions on Communications, vol. 28, no. 11, pp. 1867-1875, Oct. 1980, and J.R. Treichler et. al. in, "A New Approach to Multipath Correction of Constant Modulus Signals," IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. ASSP-31, no. 2, April 1993.

One skilled in the art would understand that the equations described herein, such as equations (3) through (5), may include scaling, change of sign, or similar constant linear modifications that are not shown for simplicity. One skilled in the art would realize that such modifications are readily determined or derived for the particular implementation. Thus, each of the described equations may be subject to such modifications, and are not limited to the exact forms presented herein.

FIG. 6 shows an exemplary embodiment of timing recovery system 600 in accordance with the present invention. As shown in FIG. 6, sample generator 601 generates a sequence of samples $y(nT + \tau)$ (equivalently, $y_n(\tau)$) of the received signal $y(t)$, where T is the sampling period and τ is the relative timing phase for generating the samples. Sample generator 601 generates samples $y_n(\tau)$ having period T and phase τ set based on a timing control signal from timing reference 602. Sample generator 601 may be an A/D converter with sampling controlled with the reference from timing reference 602. Alternatively, sample generator 601 may be a free-running A/D converter sampling at a pre-set rate, followed by an interpolator that is controlled by timing reference 602 implemented with, for example, a numerically-controlled counter.

CMA error generator 609 generates an estimate of the CMA phase error $e_{CMA}[n]$ from the sample sequence $y_n(\tau)$ using, for example, equation (5). Timing phase detector 604 determines the relative timing phase τ of the sample sequence $y_n(\tau)$ and receives CMA phase error $e_{CMA}[n]$ from CMA error generator 609. Timing phase detector 604 computes the approximate derivative of $y_n(\tau)$ with respect to τ , $dy_n(\tau)/d\tau$, and multiplies the result by the CM phase error term $e_{CMA}[n]$ to generate the approximate phase error e_τ (for example, as in equation (4')). Computation of $dy_n(\tau)/d\tau$ may be approximated using a finite impulse response (FIR) filter, such as described in

Lee and Messerschmitt, Digital Communication, Appendix 17-B, Kluwer Academic Publishers, Norwell, MA, Second Edition, 1994, which is incorporated herein by reference. Timing phase detector 604 provides the phase error e_τ (representing the CMA-weighted timing phase error) to loop filter 603 which removes high-frequency signal components and integrates the phase error 5 over time to adjust the frequency of timing reference 602, and thus ultimately sample generator 601.

Optionally, timing recovery system 600 shown in FIG. 6 may calculate one or more additional approximations of phase error in accordance with other cost criteria, such as the mean squared error (MSE) and its derivative shown in equation (2). In particular, the cost-based error 10 term, such as the error term $e_{LMS}[n]$ in equation (2'), may be calculated concurrently with the CMA error term $e_{CMA}[n]$. Signal quality may be measured and evaluated with a set of decision rules. In accordance with the decision rules, the receiver may switch between the different cost criteria to generate the timing phase error. For example, blind adaptation in accordance with 15 phase error e_τ calculated with $e_{CMA}[n]$ may be used when the receiver has a relatively low level of confidence in estimates for transmitted symbols. The receiver may then switch to a phase error e_τ calculated with $e_{LMS}[n]$ or a decision-directed version when the confidence level in estimates for transmitted symbols exceeds a certain threshold.

The least mean squares (LMS) algorithm, and specifically the decision-directed LMS algorithm, is a well-known adaptive algorithm. The LMS algorithm may generally be 20 implemented with occasional or periodic training sequences or tones (“training”) to form the phase error term $e_{LMS}[n]$. However, embodiments of the present invention may employ a blind steering mechanism whereby training may be replaced with “best estimates” of transmitted symbols. Such techniques for decision-directed LMS are described in R.W. Lucky, “Techniques for Adaptive Equalization of Digital Communication Systems,” Bell Systems Technical Journal, 25 vol. 45, no. 2, pp. 255-286, Feb. 1966, which is incorporated herein by reference.

As shown in FIG. 6, timing recovery system 600 may include cost-based error generators 609-612, and signal quality processor 614. Each of the cost-based error generators 609-612 determines a corresponding cost-based error term. For example, cost-based error generator 610 may employ the LMS error term $e_{LMS}[n]$, possibly operating in a decision-directed mode. Cost-

based error generators 611-612 may generate error terms in accordance with other methods, such as by band-edge recovery as described subsequently. Signal quality processor 614 may monitor the received signal quality to determine if estimates of the symbol value are relatively accurate, based on some predetermined threshold. Signal quality processor 614 may thus generate a signal 5 quality measure (SQM) that is provided to timing phase detector 604. Using SQM, timing phase detector 604 selects one or more of the cost-based timing errors (e.g., $e_{CMA}[n]$ or $e_{LMS}[n]$). The selected cost-based timing errors may then be used alone or in combination by timing phase detector 604 to generate the phase error e_τ .

Signal quality measured by signal quality processor 614 may include: 1) signal-to-noise 10 ratio (SNR) for the signal output from equalizer 461 (FIG. 4); and 2) SNR for the signal from, for example, a trellis decoder of detector 424 or decoder 425. In addition, signal quality processor 614 may use criteria for symbol-by-symbol switching between CMA and LMS update modes for an adaptive equalizer, such as those described in a U.S. patent application serial no. 09/549,368, entitled "A Hybrid Soft and Hard Decision Feedback Equalizer," filed on April 14, 2000. Signal 15 quality processor 614 may also use a prescribed number of symbol intervals between signal (timing) acquisition and the beginning of symbol processing. Signal quality processor 614 may use these factors alone or in combination to generate SQM.

As would be apparent to one skilled in the art, many different implementations of the present invention are possible depending on the particular receiver design. Factors such as 20 carrier-to-baseband demodulation, sampling rate, samples used for timing recovery (e.g., input samples, interpolated input samples, or equalized samples), and noise rejection processing may combine to distribute the various functions detailed in FIG. 6 throughout the signal processing path. For example, comparing FIG. 4 and FIG. 6, the timing recovery system 600 may be embodied in timing recovery module 423, A/D converter 450 (in demodulator 422), and 25 equalizer 461 (in detector 424). The following describes exemplary implementations for the preferred embodiment shown in FIG. 6.

Equalization and timing synchronization operations may be performed in parallel by jointly minimizing a blind cost function. For example, if $y_n(\tau)$ is defined as the equalizer output sample, then the CMA error term that is used to update the equalizer coefficients is given as

before in equation (5), or $e_{CMA}[k] = y(k)(\rho^2 - |y(k)|^2)$, with $k=(nT+\tau)$. This same error term, already calculated in the adaptive equalizer, may be used to form the timing phase error by combining the error term with an approximation to the derivative of $y_n(\tau)$ with respect to τ . This error term may be included within a feedback signal shown in Fig. 4 as a dashed line signal path

5 from detector 424 to timing recovery module 423.

Error term $e_{CMA}[k]$ used to compute equalizer tap coefficients is also a function of τ when $k=(nT+\tau)$. Since the equalizer, as part of its normal operation, computes $e_{CMA}[k]$, the value for $e_{CMA}[k]$ computed by the equalizer is passed to the timing phase detector 604 (FIG. 6) as $e_{CMA}[n]$. In practice, alignment of samples with the value for $e_{CMA}[k]$ computed by the equalizer

10 may be employed to account for processing delay through the equalizer. Many implementations of adaptive equalizers calculate $dJ_{CM}/dy_n(\tau)$ in order to properly adapt the equalizer coefficients, as is described in Treichler et. al., "Practical Blind Demodulators for High-Order QAM Signals," Proceedings of the IEEE, vol. 86, no. 10, pp. 1907-1926, Oct. 1998, which is incorporated herein by reference.

15 A first exemplary implementation for timing phase detector 604 is shown in FIG. 7.

Timing phase detector 604 receives $y_n(\tau)$ from sample generator 601 (or from an equalizer output) and $e_{CMA}[n]$ from, for example, an adaptive equalizer. The top branch 701 calculates an approximation of $dy_n(\tau)/d\tau$. Since the calculation of $dy_n(\tau)/d\tau$ is a non-causal algorithm, delay slots 702 and 703 are employed to generate a two-symbol-delayed version of $y_n(\tau)$, $y_{n-2}(\tau)$. The difference between the current signal sample, $y_n(\tau)$, and the two-symbol-delayed version, $y_{n-2}(\tau)$, is generated at combiner 704. Delay 705 provides a one-symbol-delayed version of $e_{CMA}[n]$, $e_{CMA}[n-1]$, that is multiplied at multiplier 706 by the difference between the current signal sample, $y_n(\tau)$, and the two-symbol-delayed version, $y_{n-2}(\tau)$. The result from multiplier 706 is the phase error e_τ that is passed to loop filter 603.

20 A second exemplary implementation for timing phase detector 604 is shown in FIG. 8.

The second exemplary implementation receives both error terms $e_{CMA}[n]$ and $e_{LMS}[n]$ from, for example, an adaptive equalizer, and selects one of the error terms to calculate e_τ . Delay slots 802 and 803 generate the two-symbol-delayed version, $y_{n-2}(\tau)$. Approximation of $dy_n(\tau)/d\tau$ occurs at combiner 804 generating the difference between the current signal sample, $y_n(\tau)$, and the two-

symbol-delayed version, $y_{n-2}(\tau)$. Delay 805 provides $e_{CMA}[n-1]$, and delay 806 provides $e_{LMS}[n-1]$. Signal SQM is applied to multiplexer 807 to select either $e_{CMA}[n-1]$ or $e_{LMS}[n-1]$ as its output to multiplier 808. Multiplier 808 combines i) the output of multiplexer 807 (either $e_{CMA}[n-1]$ or $e_{LMS}[n-1]$) with ii) the approximation of $dy_n(\tau)/d\tau$ from combiner 804. The result from multiplier 5 808 is the phase error e_τ that is passed to loop filter 603. Signal SQM may be updated every symbol, or over a block of symbols, and the decision of which error term ($e_{CMA}[n-1]$ or $e_{LMS}[n-1]$) selected with SQM is determined on a symbol-by-symbol basis.

The first and second exemplary implementations previously described with respect to FIGs. 7 and 8 calculate an approximation to the derivative of the CM cost function for the timing 10 phase error e_τ . Other decision-directed timing methods are known in the art. One such method is described in Mueller and Muller, Timing Recovery in Digital Synchronous Data Receivers, IEEE Transactions On Communications, vol. COM-24, no. 5, May 1976, which is incorporated herein by reference. In this method of Mueller et al., a class of phase error functions is presented. The example of phase error function most commonly in use is the timing phase detector function 15 given in equation (7):

$$e_\tau = y_n(\tau)s_{n-1} - y_{n-1}(\tau)s_n \quad (7)$$

The inventors have adapted the methods of Mueller et al. to make use of the CM criterion.

Equation (7) may be rewritten as in equation (7'):

$$e_\tau = e_{LMS}[n]y_{n-1}(\tau) - e_{LMS}[n-1]y_n(\tau). \quad (7')$$

20 Substituting the error term e_{CMA} for the error term e_{LMS} in equation (7'), yields a CM cost weighted calculation for e_τ as in equation (8):

$$e_\tau = e_{CMA}[n]y_{n-1}(\tau) - e_{CMA}[n-1]y_n(\tau) \quad (8)$$

A similar substitution of e_{CMA} for e_{LMS} may be done for other phase detector functions of Mueller and Muller.

25 FIG. 9 shows a third exemplary implementation for timing phase detector 604 implementing the calculation for e_τ using the CMA error term $e_{CMA}[n]$ as in equation (8). Both $y_n(\tau)$ and $e_{CMA}[n]$ are delayed by one sample in corresponding delays 901 and 902. Multiplier 903 receives $e_{CMA}[n]$ and $y_{n-1}(\tau)$ and generates $e_{CMA}[n]y_{n-1}(\tau)$. Multiplier 904 receives $e_{CMA}[n-1]$

and $y_n(\tau)$ and generates $e_{CMA}[n-1]y_n(\tau)$. Combiner 905 generates the difference between the outputs of multipliers 903 and 904 to calculate e_τ as given in equation (8).

FIG. 10 shows a fourth exemplary implementation for timing phase detector 604 that employs the signal SQM to select either i) $e_{CMA}[n]$ to calculate e_τ as given in equation (8) or ii) 5 $e_{LMS}[n]$ to calculate e_τ as given in equation (7'). This fourth exemplary implementation is similar to that shown in FIG. 9, but includes a multiplexer 1001 that selects either $e_{CMA}[n]$ or $e_{LMS}[n]$ via signal SQM. Timing phase detector 604 may select either $e_{CMA}[n]$ or $e_{LMS}[n]$ on a symbol-by-symbol basis.

As would be apparent to one skilled in the art, the first through fourth exemplary 10 implementations may be combined in various ways, with several different blind cost weighted error term calculations performed. For these different combinations, the particular calculation for error term e_τ that is actually provided to the loop filter is determined via signal SQM.

In addition, the second and fourth exemplary embodiments may be modified from using 15 $e_{CMA}[n]$ and $e_{LMS}[n]$ independently to using a weighted combination of $e_{CMA}[n]$ and $e_{LMS}[n]$. For example a linear combination of $e_{CMA}[n]$ and $e_{LMS}[n]$ may be employed to calculate phase error e_τ . Each of the multiplexers 807 and 1001 shown in FIGs. 8 or 10, respectively, may be replaced with weighted averaging mechanism 1100 shown in FIG. 11. A weighting constant, w , is determined by, for example, signal quality processor 614 (FIG. 6). As shown in FIG. 11, $e_{CMA}[n]$ is weighted by w in multiplier 1101 and $e_{LMS}[n]$ is weighted by $(1-w)$ in multiplier 1102. The 20 output values of multipliers 1101 and 1102 are added in combiner 1103 to generate a weighted phase error term $e_w[n]$. For FIG. 11, the weighting constant w may be implemented as a value that is greater than or equal to zero and less than or equal to one. If the value of w is set equal to zero or one, the operation of weighted averaging mechanism 1100 is equivalent to the operation 25 of multiplexers 807 and 1001. One skilled in the art may extend the circuit of FIG. 11 to other weighting mechanisms.

The embodiments of the present invention are described having nearly instantaneous values of error terms, or combinations of error terms. One skilled in the art would recognize that an instantaneous error term may be replaced by a filtered version of the error term, where the filtered version is generated with a filter having memory. For example, an average of past error

terms may be combined with an approximate derivative to form a timing phase error signal. This filtering may also include a weighted linear combination of error terms from different sampling instances.

Other methods of timing recovery that are not based on a gradient strategy are well

5 known in the art. For example, U.S. Patent no. 5,872,815, entitled “Apparatus for generating timing signals for a digital television receiver,” issued February 16, 1999, describes a method of band edge extraction to generate a timing phase error signal. This and related methods are usually used to begin processing from a cold start. Previously described exemplary embodiments of the present invention employ SQM and adaptive control circuitry to select an error term
10 among gradient-based choices. The present invention may be extended to select among timing phase error signals based on gradient strategies and timing phase error signals based on non-gradient based (e.g., band edge or related) strategies. FIG. 13 shows a fifth exemplary implementation of the timing phase detector of FIG. 6 switching between gradient-based and non-gradient based timing phase error detectors. For example, as illustrated in FIG. 13, SQM is
15 used by controlling multiplexer 1320 to select between a gradient-based error term generated by phase detector 1310 and a non-gradient-based error term generated by band edge timing phase detector 1330.

The first through fourth exemplary embodiments have been described with respect to timing phase detection and calculation of phase error e_τ for systems employing, for example,
20 QAM modulation. However, one skilled in the art would realize that systems based on other types of modulation may require modification to the exemplary implementations. For example, for vestigial sideband modulation (VSB) systems, a VSB signal with carrier frequency $\omega_0 \pm (\pi/2)$ and with identical pulse and vestigial spectral shaping is equivalent to a staggered quadrature amplitude modulation (SQAM) with symbol spacing of period $T_s/2$. Thus, if symbol timing of
25 the VSB signal is substantially intact, it may be converted to an SQAM signal by shifting its carrier frequency by $(F_s/4)$, where F_s is symbol frequency. The SQAM signal may then be converted to a QAM signal by delaying samples generated from the I signal component by one-half symbol period. For embodiments of the present invention designed for VSB systems, the circuit generating phase error e_τ substitutes two delay elements for each existing delay element in
30 the first through fourth exemplary implementations to select symbols separated in time by period

T. FIG. 12 illustrates modification of the first exemplary implementation of FIG. 7 adapted for use with VSB systems.

For VSB signals, a Single-Axis CM (SA-CM) criterion may be formed, as described in a paper by Shah et al, "Global convergence of a single-axis constant modulus algorithm,"

5 Proceedings of the Statistical Signal and Array Processing Workshop, Pocono Manor, PA, August 2000, which is incorporated herein by reference, and also US Patent application no. 09/100,705, entitled "A reduced complexity blind equalizer for multi-mode signaling," filed on June 19, 1998. The SA-CM criterion is based on real-part extraction of the equalizer output, and may be used for both real or complex-valued equalizer coefficients operating on both baseband 10 or passband signals or sampled signals. A single-axis stochastic gradient derived from the SA-CM criterion (SA-CMA) may also be employed. For timing recovery, the methods of deriving a phase error signal using the CM criterion described herein may also use the SA-CM criterion and SA-CMA algorithm for VSB signals. Defining $y_n(\tau)$ as the complex-valued equalizer output, then the SA-CM criterion is given by

15
$$J_{CM} = E[(\rho^2 - \text{Re}\{y_n(\tau)\}^2)^2]$$

and the SA-CM error term is given by

$$e_{SA-CMA}[n] = \text{Re}\{y_n(\tau)\}(\rho^2 - \text{Re}\{y_n(\tau)\}^2)$$

where $\text{Re}\{\bullet\}$ denotes real-part extraction.

Since the CM error term is a cubic function of the signal $y_n(\tau)$, current implementations 20 using finite precision may require large bit-width multipliers and accumulators. To reduce the computational burden, low-complexity implementations of the CMA error term have been developed, such as those described in US patent application serial no. 09/231,888, entitled "Adaptive Equalizer with Enhanced Error Quantization," filed Jan. 14, 1999, and US patent application serial no. 09/261,843 "Adaptive Equalizer with Decision Directed Constant Modulus 25 Algorithm, filed March 1999, whose teachings are incorporated herein by reference. As would be apparent to one skilled in the art, the various methods of timing recovery described herein can be extended to include the use of quantized versions of blind error terms.

To reduce stochastic jitter, the CM error term may be modified by jointly equalizing and decoding. US patent application serial no. 9/099,730 entitled “A Method of Estimating Trellis Encoded Symbols Utilizing Simplified Trellis Decoding,” filed on November 20, 2000, and US patent application entitled “Data Slicer for Combined Trellis Decoding and Equalization,” filed 5 filed on November 20, 2000, describe various decision devices which perform partial trellis decoding and which are incorporated herein by reference. For a VSB-modulated signal in accordance with the ATSC standard (an “ATSC 8-VSB signal”), the 8-level signal is decomposed into disjoint sets, each set with less than 8 elements (a “set-partitioned signal”). US patent application serial no. 09/457,496, entitled “Equalizer method and apparatus using blind 10 CMA equalization and partial decoding,” filed October 9, 1999, and incorporated herein by reference, employs the set-partitioned signals to define a new CM error term that incurs less stochastic jitter than the original CM error term. It is understood by one skilled in the art that such modified CM error terms may be employed in forming a phase error signal in accordance with the present invention.

15 Although the present invention is described herein with respect to the CM cost function, the present invention is not so limited and may be extended in general to timing phase recovery based on blind cost criteria as a class. For example, the Godard cost function G (which generalizes the CM cost function J_{CM}) is given by equation (9)

$$G = E[|y_n(\tau)|^p - \rho^p|^q], \quad (9)$$

20 which leads to an update error term $e[n]$ of the form given in equation (10):

$$e[n] = y_n(\tau) |y_n(\tau)|^{(p-2)} | |y_n(\tau)|^p - \rho^p |^{(q-1)} \text{ sign}(|y_n(\tau)|^p - \rho^p) \quad (10)$$

Furthermore, the Bussgang class of blind cost functions admit an update error term $e[n]$ of the form given in equation (11):

$$e[n] = (y_n(\tau) - g(y_n(\tau))), \quad (11)$$

25 where $g(\bullet)$ is a non-linear, memoryless function. Such blind cost functions are described in Simon Haykin, Blind Deconvolution, Chapter 2, Prentice Hall, Englewood Cliffs, NJ, 1994, whose teachings are incorporated herein by reference. For example, for the CM criterion, $g(y_n(\tau)) = (y_n(\tau)^3)/\rho^2$ is a scalar multiple of the error term $e_{CMA}[n]$ derived above. Other error

terms include, but are not limited to, 1) the Sato cost function in which $g(y_n(\tau)) = \gamma \text{ (sign } (y_n(\tau))\text{)},$ where γ is a scalar value and 2) the Benveniste-Goursat-Ruget cost function, in which $g(y_n(\tau)) = -p(y_n(\tau))/p'(y_n(\tau)),$ where $p(\bullet)$ denotes the probability density function of the variable $y_n(\tau)$ and $p'(\bullet)$ denotes the derivative of the probability density function with respect to the variable $y_n(\tau).$

5 Another class of cost functions related to the Bussgang class, but not strictly inclusive, may be represented by the weighted sum of even moments of the observation, $y_n(\tau),$ expressed as

$$J = \sum_k \lambda_k E[|y|^{2k}].$$

10 The present invention has been described with respect to a digital television receiver, and in particular, to digital television signals transmitted as symbols with m -ary quadrature amplitude modulation (QAM), an m -ary phase-shift keyed modulation (m -ary PSK), or a vestigial sideband modulation (VSB). However, one skilled in the art would realize that the techniques described herein may be applied to any receiver processing QAM, m -ary PSK, or VSB signals.

15 As would be apparent to one skilled in the art, the various functions of a timing recovery module and the timing phase detector may be implemented with circuit elements or may also be implemented in the digital domain as processing steps in a software program. Such software may be employed in, for example, a digital signal processor, micro-controller, or general-purpose computer.

20 The present invention can be embodied in the form of methods and apparatuses for practicing those methods. The present invention can also be embodied in the form of program code embodied in tangible media, such as floppy diskettes, CD-ROMs, hard drives, or any other machine-readable storage medium, wherein, when the program code is loaded into and executed by a machine, such as a computer, the machine becomes an apparatus for practicing the invention. The present invention can also be embodied in the form of program code, for example, whether stored in a storage medium, loaded into and/or executed by a machine, or 25 transmitted over some transmission medium, such as over electrical wiring or cabling, through fiber optics, or via electromagnetic radiation, wherein, when the program code is loaded into and executed by a machine, such as a computer, the machine becomes an apparatus for practicing the invention. When implemented on a general-purpose processor, the program code segments

combine with the processor to provide a unique device that operates analogously to specific logic circuits.

It will be further understood that various changes in the details, materials, and arrangements of the parts which have been described and illustrated in order to explain the nature 5 of this invention may be made by those skilled in the art without departing from the principle and scope of the invention as expressed in the following claims.